New Superconformal Chern-Simons Theories

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Kazuo Hosomichi, KML, Sangmin Lee, Sungjay Lee, Jaemo Park

M2, D3, M5 Branes

Maximally Supersymmetric Conformal Field Theory

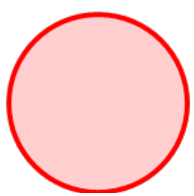
M2	2+1 dim	SO(8)	AdS4 x S7	Lie group	$N^{\frac{3}{2}}$
D3	3+1 dim	SO(6)	AdS5 x S5	Lie group	N^2
M5	5+1 dim (2,0)	SO(5)	AdS7 x S4	ADE group	N^3

Strong Coupling Limit of Type IIA Theory = M Theory

D2 Brane -> M2 Brane

$$\Phi_I, I = 1, 2, 3 \dots 7$$

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{tr}\Big(F_{\mu\nu}F^{\mu\nu} - 2D_\mu\Phi_I D^\mu\Phi_I + \dots\Big)$$



$$\mathrm{length} \sim \frac{1}{g^2}$$

$$g^2 \sim \frac{R_{11}^2}{\ell_P^3}$$

Superconformal Chern-Simons Theories as Theories on M2 Branes

- Bagger-Lambert, Gustavsson
  SU(2)xSU(2), SO(8)R symmetry $\mathcal{N} = 8$
- Aharony-Bergman-Jafferis-Maldacena
  U(N)xU(N), Chern-Simons level k

$\mathcal{N} = 6$

$\boxed{\mathbb{C}^4/\mathbb{Z}_k}$ $\boxed{k = 1, 2}$

N=3 Superconformal Theories

- Arbitrary Gauge Group
- Arbitrary Representation
- Parity Broken
- U(1)xU(1): N=4 Theory

  Kapustin and Strassler (1999)

Spin
- 1 : 1
- $\frac{1}{2}$: 3
- 0 : 3
- $-\frac{1}{2}$: 1
- -1 : 0

$$\mathcal{L} \sim kAdA + (d\phi)^2 + \psi d\psi + \frac{1}{k}\phi^2\psi^2 + \frac{1}{k^2}\phi^6$$

N=3 to N=4,5,6,8

- Gaiotto-Witten: (HLLLP)

  N=1 + SO(3) global => SO(4) R

  +twisted hyper +… => 4,5,6,8

- ABJM: (Schnabl, Tachikawa)

  N=3 => 4,5,6,8

- Deformed 3-Algebra: BL => N=6

3-dim Gaiotto-Witten Theory

- Start with N=1 Theory
- Matter: hypermultiplet $(q_\alpha^A, \psi_{\dot\alpha}^A)$
- SO(3) global symmetry:
  matter in 2-dim representation
- Gauge Groups: a subgroup of Sp(2n)

$$\omega_{AB} \qquad (t^m)^A{}_B$$

Require SO(3) => SU(2)xSU(2) R

- Write the most general N=1 superconformal theory with SO(3) symmetry

$$((A_m)_\mu, \chi_m) \qquad (q^A_\alpha, \psi^A_{\dot\alpha}, F^A_\alpha)$$

- Chern-Simons Term

$$\mathcal{L}_{\rm CS} = \frac{\varepsilon^{\mu\nu\lambda}}{4\pi} k_{mn} A^m_\mu \partial_\nu A^n_\lambda + \frac{\varepsilon^{\mu\nu\lambda}}{12\pi} f_{mnp} A^m_\mu A^n_\nu A^p_\lambda + \frac{ik_{mn}}{4\pi}\chi^m\chi^n,$$

- Matter Lagrangain

$$\begin{aligned}\mathcal{L}_{\rm kin} &= \frac{1}{2}\left(-D\bar{q}^\alpha_A D q^A_\alpha + \bar{F}^\alpha_A F^A_\alpha + i\bar\psi^{\dot\alpha}_A \not{D}\psi^A_{\dot\alpha} - i\bar\psi^{\dot\alpha}_A \chi^A{}_B q^B_\alpha + i\bar{q}^\alpha_A \chi^A{}_B \psi^B_{\dot\alpha}\right) \\ &= \frac{1}{2}\omega_{AB}\left[\epsilon^{\alpha\beta}\left(-Dq^A_\alpha D q^B_\beta + F^A_\alpha F^B_\beta\right) + i\epsilon^{\dot\alpha\dot\beta}\psi^A_{\dot\alpha}\not{D}\psi^B_{\dot\beta}\right] - i\epsilon^{\dot\alpha\beta}\psi^A_{\dot\alpha}\chi_{AB} q^B_\beta,\end{aligned}$$

- Superpotential W

$$W = \frac{\pi}{4} T_{AB,CD} \epsilon^{\alpha\beta} \epsilon^{\gamma\delta} q^A_\alpha q^B_\beta q^C_\gamma q^D_\delta .$$

- W- Lagrangian

$$-\pi T_{AB,CD} \left(\epsilon^{\alpha\beta} \epsilon^{\gamma\delta} F^A_\alpha q^B_\beta q^C_\gamma q^D_\delta + \frac{i}{2} \epsilon^{\dot\alpha\dot\beta} \epsilon^{\gamma\delta} \psi^A_{\dot\alpha} \psi^B_{\dot\beta} q^C_\gamma q^D_\delta + i \epsilon^{\dot\alpha\beta} \epsilon^{\dot\gamma\delta} \psi^A_{\dot\alpha} q^B_\beta \psi^C_{\dot\gamma} q^D_\delta \right)$$

- Integrate over gaugino & SO(4)-violating terms is

$$-i\pi q^A_\alpha q^B_\beta \psi^C_{\dot\gamma} \psi^D_{\dot\delta} \left(\boxed{k_{mn} t^m_{AC} t^n_{BD} \epsilon^{\alpha\dot\gamma} \epsilon^{\beta\dot\delta}} + \frac{1}{2} T_{AB,CD} \epsilon^{\alpha\beta} \epsilon^{\dot\gamma\dot\delta} + \boxed{T_{AC,BD} \epsilon^{\alpha\dot\gamma} \epsilon^{\beta\dot\delta}} \right)$$

- SO(4) R-Symmetry

$$T_{AC,BD} + T_{BC,AD} + k_{mn} \left(t^m_{AC} t^n_{BD} + t^m_{BC} t^n_{AD} \right) = 0.$$

- Fundamental Identity:
  constraints on gauge generators

$$k_{mn} t^m_{(AB} t^n_{C)D} = 0,$$

- Coupling constants:

$$T_{AB,CD} \;=\; \frac{1}{3} k_{mn} \left(t^m_{AC} t^n_{BD} - t^m_{BC} t^n_{AD} \right)$$

- SU(2)xSU(2) R-symmetry:

$$\delta\phi^A_\alpha = i\eta\psi^A_{\dot\alpha} \rightarrow \delta\phi^A_\alpha = i\eta_\alpha{}^{\dot\alpha}\psi^A_{\dot\alpha}$$

N=4 Lagrangian and Supersymmetric Transformation

$$\mathcal{L} = \frac{\varepsilon^{\mu\nu\lambda}}{4\pi}\left(k_{mn}A_\mu^m\partial_\nu A_\lambda^n + \frac{1}{3}f_{mnp}A_\mu^m A_\nu^n A_\lambda^p\right)$$
$$+\frac{1}{2}\omega_{AB}\left(-\epsilon^{\alpha\beta}Dq_\alpha^A Dq_\beta^B + i\epsilon^{\dot\alpha\dot\beta}\psi_{\dot\alpha}^A \not{D}\psi_{\dot\beta}^B\right)$$
$$-i\pi k_{mn}\epsilon^{\alpha\beta}\epsilon^{\dot\gamma\dot\delta}j^m_{\alpha\dot\gamma}j^n_{\beta\dot\delta} - \frac{\pi^2}{6}f_{mnp}(\mu^m)^\alpha{}_\beta(\mu^n)^\beta{}_\gamma(\mu^p)^\gamma{}_\alpha$$

$$\mu^m_{\alpha\beta} = t^m_{AB}q^A_\alpha q^B_\beta$$

$$j^m_{\alpha\dot\gamma} \equiv q^A_\alpha t^m_{AC}\psi^C_{\dot\gamma}$$

$$\delta q^A_\alpha = i\eta_\alpha{}^{\dot\alpha}\psi^A_{\dot\alpha}, \qquad \delta A^m_\mu = 2\pi i\eta^{\alpha\dot\alpha}\gamma_\mu j^m_{\alpha\dot\alpha},$$
$$\delta\psi^A_{\dot\alpha} = \left[\not{D}q^A_\alpha + \frac{2\pi}{3}(t_m)^A{}_B q^B_\beta(\mu^m)^\beta{}_\alpha\right]\eta^\alpha{}_{\dot\alpha}.$$

Lie Super-Algebra

$$[M^m, M^n] = f^{mn}{}_p M^p$$

$$[M^m, Q_A] = Q_B (t^m)^B{}_A$$

$$\{Q_A, Q_B\} = t^m_{AB} M_m$$

Jacobi Identity -> Fundamental Identity

Gauge Group and Matter Representation

Classification of Super Lie Algebra

- U(N)xU(M)
- O(N)xSp(2M)
- SO(7)xSp(2) 8-dim spinor
- G2xSp(2)
- SU(2)xSU(2)xSU(2)
- SU(N), adjoint

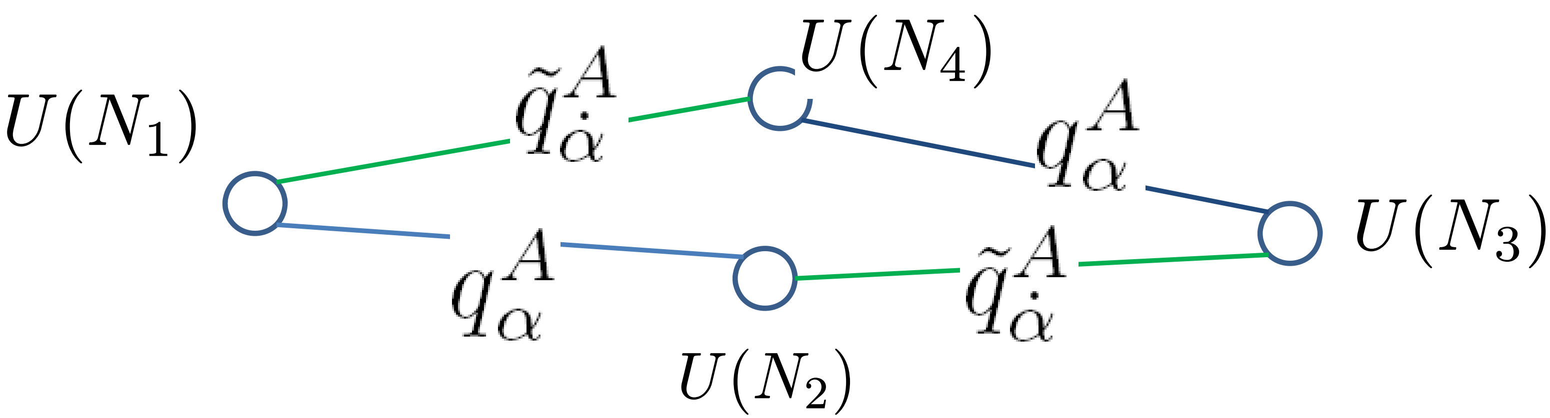
- The Chern-Simons level k, -k

• Our Generalization

- Twisted Hyper-Multiplet $\boxed{\tilde{q}^A_{\dot{\alpha}}, \tilde{\psi}^A_{\alpha}}$

$$\tilde{\mu}^m_{\dot{\alpha}\dot{\beta}} \equiv \tilde{t}^m_{AB}\tilde{q}^A_{\dot{\alpha}}\tilde{q}^B_{\dot{\beta}} \qquad \tilde{j}^m_{\dot{\alpha}\alpha} \equiv \tilde{q}^A_{\dot{\alpha}}\tilde{t}^m_{AB}\tilde{\psi}^B_{\alpha}$$

- Linear Quiver Type Theory



$$
\begin{aligned}
\mathcal{L} = & \frac{\varepsilon^{\mu\nu\lambda}}{4\pi}\left(k_{mn}A^m_\mu \partial_\nu A^n_\lambda + \frac{1}{3} f_{mnp} A^m_\mu A^n_\nu A^p_\lambda\right) \\
& + \frac{1}{2}\omega_{AB}\left(-\epsilon^{\alpha\beta} D q^A_\alpha D q^B_\beta + i\epsilon^{\dot\alpha\dot\beta}\psi^A_{\dot\alpha} \not{D} \psi^B_{\dot\beta}\right) + \frac{1}{2}\omega_{AB}\left(-\epsilon^{\dot\alpha\dot\beta} D\tilde{q}^A_{\dot\alpha} D\tilde{q}^B_{\dot\beta} + i\epsilon^{\alpha\beta}\tilde{\psi}^A_\alpha \not{D}\tilde{\psi}^B_\beta\right) \\
& - i\pi k_{mn}\epsilon^{\alpha\beta}\epsilon^{\dot\gamma\dot\delta} j^m_{\alpha\dot\gamma} j^n_{\beta\dot\delta} - i\pi k_{mn}\epsilon^{\dot\alpha\dot\beta}\epsilon^{\gamma\delta}\tilde{j}^m_{\dot\alpha\gamma}\tilde{j}^n_{\dot\beta\delta} + 4\pi i k_{mn}\epsilon^{\alpha\gamma}\epsilon^{\dot\beta\dot\delta} j^m_{\alpha\dot\beta}\tilde{j}^n_{\dot\delta\gamma} \\
& + i\pi k_{mn}\left(\epsilon^{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\tilde{\mu}^m_{\dot\alpha\dot\beta}\psi^A_{\dot\gamma} t^n_{AB}\psi^B_{\dot\delta} + \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\mu^m_{\alpha\beta}\tilde{\psi}^A_\gamma \tilde{t}^n_{AB}\tilde{\psi}^B_\delta\right) \\
& - \frac{\pi^2}{6} f_{mnp}(\mu^m)^\alpha{}_\beta (\mu^n)^\beta{}_\gamma (\mu^p)^\gamma{}_\alpha - \frac{\pi^2}{6} f_{mnp}(\tilde{\mu}^m)^{\dot\alpha}{}_{\dot\beta}(\tilde{\mu}^n)^{\dot\beta}{}_{\dot\gamma}(\tilde{\mu}^p)^{\dot\gamma}{}_{\dot\alpha} \\
& + \pi^2 (\tilde{\mu}^{mn})^{\dot\gamma}{}_{\dot\gamma}(\mu_m)^\alpha{}_\beta(\mu_n)^\beta{}_\alpha + \pi^2(\mu^{mn})^\gamma{}_\gamma(\tilde{\mu}_m)^{\dot\alpha}{}_{\dot\beta}(\tilde{\mu}_n)^{\dot\beta}{}_{\dot\alpha}\,. \qquad (2.4)
\end{aligned}
$$

$$
\begin{aligned}
\delta q^A_\alpha &= + i\eta_\alpha{}^{\dot\alpha}\psi^A_{\dot\alpha}, \quad \delta\tilde{q}^A_{\dot\alpha} = -i\eta_{\dot\alpha}{}^\alpha\tilde{\psi}^A_\alpha, \quad \delta A^m_\mu = 2\pi i \eta^{\alpha\dot\alpha}\gamma_\mu (j^m_{\alpha\dot\alpha} - \tilde{j}^m_{\dot\alpha\alpha}), \\
\delta\psi^A_{\dot\alpha} &= + \left[\not{D} q^A_\alpha + \frac{2\pi}{3}(t_m)^A{}_B q^B_\beta (\mu^m)^\beta{}_\alpha\right]\eta^\alpha{}_{\dot\alpha} - 2\pi (t_m)^A{}_B q^B_\beta (\tilde{\mu}^m)^{\dot\beta}{}_{\dot\alpha}\eta^\beta{}_{\dot\beta} \\
\delta\tilde{\psi}^A_\alpha &= - \left[\not{D}\tilde{q}^A_{\dot\alpha} + \frac{2\pi}{3}(\tilde{t}_m)^A{}_B \tilde{q}^B_{\dot\beta}(\tilde{\mu}^m)^{\dot\beta}{}_{\dot\alpha}\right]\eta_\alpha{}^{\dot\alpha} + 2\pi(\tilde{t}_m)^A{}_B\tilde{q}^B_{\dot\beta}(\mu^m)^\beta{}_\alpha \eta_\beta{}^{\dot\beta}.
\end{aligned}
$$

SU(2)xSU(2) gauge group: BLG Theory of N=8

Mass deformation

$$-\omega_{AB}\left(\frac{m^2}{2}\epsilon^{\alpha\beta}q^A_\alpha q^B_\beta + \frac{m^2}{2}\epsilon^{\dot\alpha\dot\beta}\tilde q^A_{\dot\alpha}\tilde q^B_{\dot\beta} + \frac{i}{2}m\epsilon^{\dot\alpha\dot\beta}\psi^A_{\dot\alpha}\psi^B_{\dot\beta} - \frac{i}{2}m\epsilon^{\alpha\beta}\tilde\psi^A_\alpha\tilde\psi^B_\beta\right)$$
$$-\frac{2\pi}{3}mk_{mn}\left((\mu^m)_{\alpha\beta}(\mu^n)^{\beta\alpha} - (\tilde\mu^m)_{\dot\alpha\dot\beta}(\tilde\mu^n)^{\dot\beta\dot\alpha}\right),$$

$$\delta_{\mathrm{mass}}\psi^A_{\dot\alpha} = mq^A_\alpha\eta^\alpha{}_{\dot\alpha}, \qquad \delta_{\mathrm{mass}}\tilde\psi^A_\alpha = m\tilde q^A_{\dot\alpha}\eta_\alpha{}^{\dot\alpha}.$$

N=5 SCFT

$$t^m_{AB} = \tilde{t}^m_{AB}$$

$G_1 \times G_2$ Two Node System

$$\Phi^A_\alpha = \begin{pmatrix} q^A_\alpha \\ \tilde{q}^A_{\dot\alpha} \end{pmatrix}, \qquad \Psi^A_\alpha = \begin{pmatrix} \tilde{\psi}^A_\alpha \\ \psi^A_{\dot\alpha} \end{pmatrix}$$

SU(2)xSU(2) => Sp(4) R-symmetry

$$C^{\alpha\beta} = \begin{pmatrix} \epsilon^{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \qquad \mathcal{M}^m_{\alpha\beta} \equiv t^m_{AB}\Phi^A_\alpha\Phi^B_\beta \quad \mathcal{J}^m_{\alpha\beta} \equiv t^m_{AB}\Phi^A_\alpha\Psi^B_\beta$$

$$\begin{aligned}\mathcal{L} = &\frac{\varepsilon^{\mu\nu\lambda}}{4\pi}\left(k_{mn}A^m_m\partial_\nu A^n_\lambda + \frac{1}{3}f_{mnp}A^m_\mu A^n_\nu A^p_\lambda\right) \\ &+\frac{1}{2}\omega_{AB}C^{\alpha\beta}\left(-D\Phi^A_\alpha D\Phi^B_\beta + i\Psi^A_\alpha \not{D}\Psi^B_\beta\right) - i\pi k_{mn}C^{\alpha\beta}C^{\gamma\delta}\left(\mathcal{J}^m_{\alpha\gamma}\mathcal{J}^n_{\beta\delta} - 2\mathcal{J}^m_{\alpha\gamma}\mathcal{J}^n_{\delta\beta}\right) \\ &+\frac{2\pi^2}{15}f_{mnp}(\mathcal{M}^m)^\alpha{}_\beta(\mathcal{M}^n)^\beta{}_\gamma(\mathcal{M}^p)^\gamma{}_\alpha + \frac{3\pi^2}{5}(\mathcal{M}^{mn})^\gamma{}_\gamma(\mathcal{M}_m)^\alpha{}_\beta(\mathcal{M}_n)^\beta{}_\alpha, \qquad (2.13)\end{aligned}$$

$$\delta\Phi^A_\alpha = i\eta_\alpha{}^\beta\Psi^A_\beta, \quad \delta A^m_\mu = 2\pi i\eta^{\alpha\beta}\gamma_\mu\mathcal{J}^m_{\alpha\beta},$$

$$\delta\Psi^A_\alpha = \left[\not{D}\Phi^A_\gamma + \frac{2\pi}{3}(t_m)^A{}_B\Phi^B_\beta(\mathcal{M}^m)^\beta{}_\gamma\right]\eta^\gamma{}_\alpha - \frac{4\pi}{3}(t_m)^A{}_B\Phi^B_\beta(\mathcal{M}^m)^\gamma{}_\alpha\eta^\beta{}_\gamma.$$

Sp(2M)x O(N), U(M)xU(N)

N=6 SCFT

Complex Representation

$$(t^A{}_B)_{\mathcal{N}=5} = \begin{pmatrix} t^A{}_B & 0 \\ 0 & -t_A{}^B \end{pmatrix}$$

$$(\Phi^A_\alpha)_{\mathcal{N}=5} = \begin{pmatrix} \Phi^A_\alpha \\ C_{\alpha\beta}\bar{\Phi}^\beta_A \end{pmatrix} , \qquad (\Psi^A_\alpha)_{\mathcal{N}=5} = \begin{pmatrix} C_{\alpha\beta}\Psi^{\beta A} \\ -\bar{\Psi}_{\alpha A} \end{pmatrix}$$

$$(M^m)^\alpha{}_\beta \equiv \bar{\Phi}^\alpha_A (t^m)^A{}_B \Phi^B_\beta , \quad (M^{mn})^\alpha{}_\beta \equiv \bar{\Phi}^\alpha_A (t^m t^n)^A{}_B \Phi^B_\beta$$

$$(J^m)_{\alpha\beta} \equiv \Phi^A_\alpha (t^m)_A{}^B \bar{\Psi}_{\beta B} , \quad (\bar{J}^m)^{\alpha\beta} \equiv \bar{\Phi}^\alpha_A (t^m)^A{}_B \Psi^{\beta B} .$$

Sp(4) => SU(4) R-Symmetry

$$\mathcal{L} = \frac{\varepsilon^{\mu\nu\lambda}}{4\pi}\left(k_{mn}A_m^m\partial_\nu A_\lambda^n + \frac{1}{3}f_{mnp}A_\mu^m A_\nu^n A_\lambda^p\right) - D\bar{\Phi}_A^\alpha D\Phi_\alpha^A + i\bar{\Psi}_{\alpha A}\not{D}\Psi^{\alpha A}$$
$$+i\pi\left[2(\bar{J}_m)^{\alpha\beta}(J_m)_{\alpha\beta} - 4(\bar{J}_m)^{\alpha\beta}(J^m)_{\beta\alpha} + \epsilon^{\alpha\beta\gamma\delta}(J_m)_{\alpha\beta}(J^m)_{\gamma\delta} + \epsilon_{\alpha\beta\gamma\delta}(\bar{J}_m)^{\alpha\beta}(\bar{J}^m)^{\gamma\delta}\right]$$
$$-\frac{4\pi^2}{3}f_{mnp}(M^m)^\alpha{}_\beta(M^n)^\beta{}_\gamma(M^p)^\gamma{}_\alpha + 4\pi^2(M^{mn})^\alpha{}_\beta(M_n)^\beta{}_\gamma(M_m)^\gamma{}_\alpha. \quad (3.13)$$

$$\delta\Phi_\alpha^A = -i\eta_{\alpha\beta}\Psi^{A\beta}, \quad \delta A_\mu^m = 2\pi i\left(\eta^{\alpha\beta}\gamma_\mu(J^m)_{\alpha\beta} + \eta_{\alpha\beta}\gamma_\mu(\bar{J}^m)^{\alpha\beta}\right),$$
$$\delta\Psi^{A\alpha} = \left[\not{D}\Phi_\gamma^A - \frac{2\pi}{3}(t_m)^A{}_B\Phi_\beta^B(M^m)^\beta{}_\gamma\right]\eta^{\gamma\alpha} + \frac{4\pi}{3}(t_m)^A{}_B\Phi_\beta^B(M^m)^\alpha{}_\gamma\eta^{\gamma\beta}$$
$$+\frac{2\pi}{3}\epsilon^{\alpha\beta\gamma\delta}(t_m)^A{}_B\Phi_\beta^B(M^m)^\rho{}_\gamma\eta_{\delta\rho}\,.$$

U(M)xU(N): ABJM Case

U(1)xSp(2N): Something New

U(2N)xU(2N) -> Sp(2N)xO(2N)

M2 Branes on Orbifolds

$$\mathbb{C}^4/\mathbb{Z}_k \qquad (\phi_1, \phi_2, \phi_3, \phi_4)$$

$$\alpha \ : \ \phi_i \ \longrightarrow \ e^{2\pi i/k}\phi_i .$$

$$\mathbb{C}^4/\hat{D}_{k+2}$$

$$\beta \ : \ (\phi_1, \phi_2, \phi_3, \phi_4) \ \longrightarrow \ (\phi_2^*, -\phi_1^*, \phi_4^*, -\phi_3^*),$$

$$\alpha^{2k} = 1, \qquad \beta^2 = \alpha^k, \qquad \beta\alpha\beta^{-1} = \alpha^{-1}.$$

Mass Deformation: supersymmetry preserved

- BLG: Supersymmetry Preserved,

  SO(8) R -> SO(4)x SO(4)
- Gaiotto-Witten: SO(4) R preserved
- N=5 Theories: SO(5) R -> SO(4)
- N=6 Theories: SO(6) R -> SO(4)x U(1)

Conclusion

- New Superconformal Chern-Simons Theories are found.
- They deserve further exploration.
- There may be more such theories to be discovered.